Some Weaker Forms of Decompositions In Supra Topological Spaces

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Abstract-The aim of this paper is to introduce a new decomposition which are weaker than supra locally closed sets and study their relationship. Further we introduce the different notions of generalization of continuous functions in supra topological spaces and some of their properties are discussed.

Keywords- S-SLC sets, S-GLSC sets, S-SLGC sets, S-SGLC sets, S-SL-continuous, S-GLS-continuous, S-SLG-continuous S-SGL-continuous, S-SL-irresolute S-GLS-irresolute, S-SLG-irresolute and S-SGL-irresolute.

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1. Introduction

In a topological space, Ganster and Reilly used locally closed sets in [1], to define LC-continuity and LC-irresoluteness. Balachandran et. al. [2] introduced the concept of of generalized locally closed sets and discussed some of its properties. Balachandran et. al [3] introduced new classes of sets namely, slc, sglc, lsc and glsc, which are weaker than locally closed sets and studied their relationship. Further, they defined the repective weak forms of continuity. Jin Han Park et. al. [4] introduced sglc* set and sglc** set and different notions of generalizations of continuous functions and investigated some of their properties.

In a supra topological space, Mashhour et. al. [5] introduced the concept of supra open sets which is considered as a generalization of open sets, semi-open sets and pre-open sets. Kamaraj et. al. [7] defined and studied the concepts of supra sg-closed sets and supra gs-closed sets and a new classes of spaces.

Dayana Mary et. al.[8] introduced a new class of decomposition namely supra generalized locally closed sets and new class of maps called supra generalized locally continuous functions. Next, they defined a new class of decomposition called supra regular generalized locally closed sets [9] and S-RGL-continuous functions. Also, they introduced supra β-LC sets [10] and S-β-L-continuous functions and discussed some of their properties. Furthermore, they defined supra-ω locally closed sets [11] and supra ω-locally continuous functions and investigated some of the basic properties for this class of functions.

In this paper we introduce new class of sets, each of which contains supra locally closed set. We also study the relationship among these classes, some of their properties and the respective weak forms of continuity. This paper mainly deals with the supra topological properties of supra semi generalized locally closed sets.

2. Preliminaries

Throughout this paper, (X, τ), (Y, σ) and (Z, η) (or simply, X, Y and Z) represent topological space on which no separation axioms are assumed, unless explicitly stated. For a subset A of (X, τ), the closure and interior of A in X are denoted by cl(A) and int (A), respectively. Let P(X) be the power set of X. The complement of A is denoted by X-A or A^c.

Now we recall some definitions and results which are useful in the sequel.

Definition 1 [5,12]

Let X be a non-empty set. The subfamily $\mu \subseteq P(X)$ is said to a supra topology on X if $X \in \mu$ and μ is closed under arbitrary unions. The pair (X, μ) is called a supra topological space.

The elements of μ are said to be supra open in (X, μ). Complement of supra open sets are called supra closed sets.

Definition 2 [12]

Let A be a subset (X, μ). Then

(i) The supra closure of a set A is, denoted by $cl^{\mu}(A)$, defined as $cl^{\mu}(A) = \cap \{B : B$ is a supra closed and $A \subseteq B\}$.

(ii) The supra interior of a set A is, denoted by $int^{\mu}(A)$, defined as $int^{\mu}(A) = \cup \{B : B$ is a supra open and $B \subseteq A\}$.

Definition 3 [5]

Let (X, τ) be a topological space and μ be a supra topology on X. We call μ is a supra topology associated with τ if $\tau \subseteq \mu$

Definition 4 [13]

Let (X, τ) and (Y, σ) be two topological spaces and $\tau \subseteq \mu$. A function f: $(X, \tau) \rightarrow (y,\sigma)$ is called supra continuous, if the inverse image of each open set of Y is a supra open set in X.

Definition 5 [14]

Let (X, τ) and (Y, σ) be two topological spaces and μ and λ be supra topologies associated with τ and σ respectively. A function f : $(X, \tau) \rightarrow (y,\sigma)$ is said to be supra irresolute , if f^{-1} (A) is supra open set of X for every supra open set A in Y.

Definition 6 [6]

Let A and B be subsets of X. Then A and B are said to be supra separated, if A ∩ $cl^{\mu}(B)$ = B ∩ $cl^{\mu}(A)$ = ϕ.

Definition 7 [6]

Let (X, μ) be a supra topological space. A subset A of X is called supra g-closed if $cl^{\mu}(A)$ ⊆U whenever A ⊆ U and U is supra open in X.

The complement of supra g-closed set is called supra g-open.

Definition 8 [6]

Let A be a subset (X, μ). Then

(i) The supra g-closure of a set A is, denoted by $cl_g^{\mu}(A)$, defined as $cl_g^{\mu}(A)$ = ∩ {B : B is a supra g-closed and A ⊆ B}.

(ii) The supra g-interior of a set A is, denoted by $int_g^{\mu}(A)$, defined as $int_g^{\mu}(A)$ = ∪ {B : B is a supra g-open and B ⊆ A}.

Definition 9 [5] Let (X, μ) be a supra topological space. A subset A of X is called supra semi-open, if A⊆ $cl^{\mu}(int^{\mu}(A))$.

Definition 10 [7]

Let (X, μ) be a supra topological space. A subset A of X is called supra sg-closed if $scl^{\mu}(A)$ ⊆U whenever A ⊆ U and U is supra semi-open in X.

The complement of supra sg-closed set is called supra sg-open.

Definition 11 [15]

A subset S of a supra topological space X is called supra locally closed, if S =A ∩ B, where A is supra open and B is supra closed in X.

Definition 12 [8] Let (X, μ) be a supra topological space. A subset A of (X, μ) is called supra generalized locally closed set (briefly supra g-locally closed set), if A=U ∩ V, where U is supra g-open in (X, μ) and V is supra g-closed in (X, μ).

The collection of all supra generalized locally closed sets of X will be denoted by S-GLC(X).

Remark 13 [8] Every supra g-closed set (resp. supra g-open set) is S-GLC.

Definition 14 [8] For a subset A of supra topological space (X, μ), A ∈ S-GLC*(X, μ), if there exist a supra g-open set U and a supra closed set V of (X, μ), respectively such that A=U ∩ V.

Definition 15 [8] For a subset A of (X, μ), A ∈ S-GLC**(X, μ), if there exist an supra open set U and a supra g-closed set V of (X, μ), respectively such that A=U ∩ V.

Definition 16 [8] Let (X, τ) and (Y, σ) be two topological spaces and τ ⊆ μ. A function f : (X, τ) → (Y,σ) is called S-GL - continuous (resp., S-GL* - continuous, resp., S-GL** - continuous), if f^{-1}(A) ∈ S-GLC (X,μ), (resp., f^{-1} (A) ∈ S-GLC* (X,μ), resp., f^{-1} (A)∈ S-GLC** (X,μ)) for each A ∈ σ.

3. Weak Forms of Supra Locally Closed Sets

In this section, we introduce some weak forms of supra locally closed sets and study the relations among them.

3.1 Supra Semi-Locally Closed Sets

In this we define a supra semi-locally closed sets and discuss some of their properties.

Definition 1.

Let (X, μ) be a supra topological space. A subset A of (X, μ) is called supra semi locally closed set (briefly supra slc set), if A=U ∩ V, where U is supra semi-open in (X, μ) and V is supra semi-closed in (X, μ).

The class of all supra semi locally closed sets of X will be denoted by S-SLC(X).

Remark 2.

Every supra semi-closed set (resp. supra semi-open set) is S-SLC.

Definition 3.

For a subset A of supra topological space (X, μ), A ∈ S-SLC*(X,μ), if there exist a supra semi-open set U and a supra closed set V of (X,μ), respectively such that A=U ∩ V.

Remark 4.

Every supra closed set (resp. supra semi-open set) is S-SLC*.

Definition 5.

For a subset A of (X, μ), A ∈ S-SLC**(X, μ), if there exist a supra open set U and a supra semi-closed set V of (X, μ), respectively such that A=U ∩ V.

Remark 6.

Every supra semi-closed set (resp. supra open set) is S-SLC**.

Remark 7.

(i) Intersection of two supra S-SLC set is generally not an S-SLC set.

(ii) Intersection of two supra S-SLC* set is generally not an S-SLC* set.

(iii) Intersection of two supra S-SLC** set is generally not an S-SLC** set.

Example 8.

Let X = {a, b, c, d} and μ = {ϕ, X, {a, c}, {b, c}, {a, b, c}, {a, b, d}, {b, c, d}}.

Then

(i) {a, c, d} and {b, c, d} are S-SLC but {c, d} not S-SLC.

(ii) {b, c, d} and {a, c, d} are S-SLC* but {c, d} not S-SLC*.

(iii) {a, b, c} and {a, b, d} are S-SLC** but {a, b} not S-SLC**.

Remark 9.

(i) Union of two supra S-SLC set is generally not an S-SLC set.

(ii) Union of two supra S-SLC* set is generally not an S-SLC* set.

(iii) Union of two supra S-SLC** set is generally not an S-SLC** set.

Example 10.

Let X = {a, b, c, d} and μ = {ϕ, X, {a, c}, (b, c}, {a, b, c}, {a, b, d}, {b, c, d}}.

Then

(i) {c} and {d} are S-SLC but {c, d} not S-SLC.

(ii) {a} and {b} are S-SLC* but {a, b} not S-SLC*.

(iii) {d} and {a, c} are S-SLC** but {a, c, d} not S-SLC**.

Theorem 11.

For a subset A of (X, μ), the following are equivalent:

(i) A ∈ S-SLC*(X, μ).

(ii) A = U ∩ $cl^{\mu}(A)$, for some supra semi-open set U.

(iii) $cl^{\mu}(A)$ – A is supra semi-closed.

(iv) A∪ [X- $cl^{\mu}(A)$], is supra semi-open.

Proof. (i) ⇒ (ii): Given A ∈ S-SLC*(X, μ)

Then there exist a supra semi-open subset U and a supra closed subset V such that A=U ∩V. Since A ⊂ U and A ⊂ $cl^{\mu}(A)$, A ⊂ U ∩ $cl^{\mu}(A)$.

Conversely, we have $cl^{\mu}(A)$⊂ V and hence A = U ∩ V ⊃ U ∩ $cl^{\mu}(A)$. Therefore A = U ∩ $cl^{\mu}(A)$

(ii) ⇒ (i): Let A = U ∩ $cl^{\mu}(A)$, for some supra semi-open set U. Clearly, $cl^{\mu}(A)$ is supra closed and hence A = U ∩ $cl^{\mu}(A)$ ∈S-SLC*(X, μ).

(ii) ⇒ (iii): Let A = U ∩ $cl^{\mu}(A)$, for some supra semi-open set U.
Then A ∈ S-SLC*(X, μ). This implies U is supra semi-open and $cl^{\mu}(A)$ is supra semi-closed. Therefore, $cl^{\mu}(A) -$ A is supra semi-closed.

(iii) ⇒ (ii): Let U= X – [$cl^{\mu}(A)$- A]. By (iii), U is supra semi-open in X. Then A = U ∩ $cl^{\mu}(A)$ holds.

(iii) ⇒ (iv): Let Q =$cl^{\mu}(A)$–A be supra semi-closed. Then X-Q = X - [$cl^{\mu}(A)$– A] =A ∪ [(X- $cl^{\mu}(A)$]. Since X-Q is supra semi-open, A ∪ [X- $cl^{\mu}(A)$] is supra semi-open.

(vi) ⇒ (iii): Let U = A ∪ [(X- $cl^{\mu}(A)$]. Since X – U is supra semi-closed and X - U = $cl^{\mu}(A)$ - A holds, $cl^{\mu}(A)$ - A is supra semi-closed.

Theorem 12.

For a subset A of (X, μ), the following are equivalent:

(v) A ∈ S-SLC (X, μ).

(vi) A = U ∩ $scl^{\mu}(A)$, for some supra semi-open set U.

(vii) $scl^{\mu}(A)$ – A is supra semi-closed.

(viii) A∪ [X- $scl^{\mu}(A)$], is supra semi-open.

Proof. (i) ⇒ (ii): Given A ∈ S-SLC*(X, μ)

Then there exist a supra semi-open subset U and a supra closed subset V such that A=U ∩V. Since A ⊂ U and A ⊂ $scl^{\mu}(A)$, A ⊂ U ∩ $scl^{\mu}(A)$.

Conversely, we have $scl^{\mu}(A)$⊂ V and hence A = U ∩ V ⊃ U ∩ $scl^{\mu}(A)$. Therefore A = U ∩ $scl^{\mu}(A)$

(ii) ⇒ (i): Let A = U ∩ $scl^{\mu}(A)$, for some supra semi-open set U. Clearly, $scl^{\mu}(A)$ is supra closed and hence A = U ∩ $scl^{\mu}(A)$ ∈S-SLC*(X, μ).

(ii) ⇒ (iii): Let A = U ∩ $scl^{\mu}(A)$, for some supra semi-open set U.

Then A ∈ S-SLC*(X, μ). This implies U is supra semi-open and $scl^{\mu}(A)$ is supra semi-closed. Therefore, $scl^{\mu}(A) -$ A is supra semi-closed.

(iii) ⇒ (ii): Let U= X – [$scl^{\mu}(A)$- A]. By (iii), U is supra semi-open in X. Then A = U ∩ $scl^{\mu}(A)$ holds.

(iii) ⇒ (iv): Let Q =$scl^{\mu}(A)$–A be supra semi-closed. Then X-Q = X - [$scl^{\mu}(A)$– A] =A ∪ [(X- $scl^{\mu}(A)$]. Since X-Q is supra semi-open, A ∪ [X- $cl^{\mu}(A)$] is supra semi-open.

(vi) ⇒ (iii): Let U = A ∪ [(X- $scl^{\mu}(A)$]. Since X – U is supra semi-closed and X - U = $scl^{\mu}(A)$ - A holds, $scl^{\mu}(A)$ - A is supra semi-closed.

Theorem 13.

For a subset A of (X, μ), if A ∈ S-SLC**(X, μ), then there exist an supra open set G such that A = G ∩ $cl^{\mu}(A)$.

Proof.

Let A ∈ S-SLC**(X, μ). Then there exist a supra open subset U and a supra semi-closed subset V such that A=U ∩V. Since A ⊂ U and A ⊂ $cl^{\mu}(A)$, A ⊂ U ∩ $cl^{\mu}(A)$.

Conversely, we have $cl^{\mu}(A)$⊂ V and hence A = U ∩ V ⊃ U ∩ $cl^{\mu}(A)$. Therefore A = U ∩ $cl^{\mu}(A)$.

Theorem 14.

Let A be a subset of (X, μ). Then A∈S-SLC**(X, μ) if and only if A = U ∩ $scl^{\mu}(A)$, for some supra open set U.

Proof.

Let A ∈ S-SLC**(X, μ). Then there exist a supra open subset U and a supra semi-closed subset V such that A=U ∩V. Since A ⊂ U and A ⊂ $scl^{\mu}(A)$, A ⊂ U ∩ $scl^{\mu}(A)$.

Conversely, we have $scl^{\mu}(A)$⊂ V and hence A = U ∩ V ⊃ U ∩ $scl^{\mu}(A)$. Therefore A = U ∩ $scl^{\mu}(A)$.

Theorem 15.

Let A be a subset of (X, μ). If A ∈ S-SLC**(X, μ), then $scl^{\mu}(A)$- A supra semi-closed and A ∪ [(X- $scl^{\mu}(A)$] is supra semi-open.

Proof. The proof is obvious from the definitions and results.

Remark 16.

The converse of the above theorem need not be true as seen the following example.

Example 17.

Let X = {a, b, c, d} and μ = {ϕ, X, {a}, {a, b}, {b, c}, {a, b, c}}. If A = {a, b, d}, then $scl^{\mu}(A)$ – A is supra semi-closed and A ∪ [(X - $scl^{\mu}(A]$ is supra semi-open but A ∉ S-SLC**(X, μ).

Theorem 18.

Let A and Z be subset of (X, μ) and let A ⊆ Z. If Z is supra semi-open in (X, μ) and A ∈ S-SLC* (Z, μ/Z), then A ∈ S-SLC* (X, μ).

Proof. Suppose A ∈ S-SLC*, then there exist a supra semi-open set H of (Z, μ/Z) such that A = H ∩ $cl_Z^{\mu}(A)$. But $cl_Z^{\mu}(A)$ = Z ∩ $cl^{\mu}(A)$. Therefore A = H ∩ Z ∩ $cl^{\mu}(A)$, where H ∩ Z is supra semi-open. Thus A ∈ S-SLC* (X, μ).

Remark 19.

The following example shows that the assumption that Z is supra semi-open cannot be removed from the above theorem.

Example 20.

Let X = {a, b, c, d}, μ = {ϕ, X, {a, c}, {b, c}, {a, b, c}, {a, b, d}, {b, c, d}}. Let V be the collection of all supra semi-open sets of (X, μ). Then V = {ϕ, X, {a, c}, {b, c}, {a, b, c}, {a, b, d}, {a, c, d}, {b, c, d}}. Put Z = A = {a, b}. Then Z is not supra semi-open and A∈ S-SLC* (Z, μ/Z). However A∉ S-SLC* (X, μ), since S-SLC* (X, μ) = P(X) – {a, b} and {c, d}.

Theorem 21.

Let A and Z be subset of (X, μ) and let A ⊆ Z. If Z is supra open in (X, μ) and A ∈ S-SLC** (Z, μ/Z), then A ∈ S-SLC**(X, μ).

Proof.

By theorem 14, there exist a supra open set H of (Z, μ/Z) such that A = H ∩ $scl_Z^{\mu}(A)$. But $scl_Z^{\mu}(A)$ = Z ∩ $scl^{\mu}(A)$. Therefore A = H ∩ Z ∩ $scl^{\mu}(A)$, where H ∩ Z is supra open. Thus A ∈ S-SLC** (X, μ).

Theorem 22.

Let A and Z be subset of (X, μ) and let A ⊆ Z. If Z is supra semi-closed in (X, μ) and A ∈ S-SLC**(Z, μ/Z), then A ∈ S-SLC**(X, μ).

Proof.

Let Z be supra semi-closed in (X, μ) and A ∈ S-SLC**(Z, μ/Z). Then there exists a supra open set U in (Z, μ/Z) and a supra semi-closed set V in (Z, μ/Z) such that A = U ∩ V. As U is supra open in (Z, μ/Z), there exists a supra open set G in (X, μ) such that U = G ∩ Z. As V is supra semi-closed in (Z, μ/Z) and Z is supra semi-closed in X. Then V supra semi-closed in (X, μ). Now A= U ∩ V= (G ∩ Z) ∩ V = G ∩ (Z ∩ V) = G ∩ V, where G is supra open in (X, μ) and V is supra semi-closed in (X, μ). Hence A ∈ S-SLC**(X, μ).

Theorem 23.

Let (X, μ) and (Y, λ) be the supra topological spaces.

(1) If M ∈ S-SLC(X, μ) and N ∈ S-SLC(Y, λ), then M × N∈S-SLC(X × Y, μ × λ).

(2) If M ∈ S-SLC*(X, μ) and N ∈ S-SLC*(Y, λ), then M × N∈S-SLC*(X × Y, μ × λ).

(3) If M ∈ S-SLC**(X, μ) and N ∈ S-SLC**(Y, λ), then M × N ∈ S-SLC**(X ×Y, μ × λ).

Proof. Let M ∈ S-SLC(X, μ) and N ∈ S-SLC(Y, λ). Then there exist a supra semi-open sets P and P′ of (X, μ) and (Y, λ) and supra semi-closed sets Q and Q′ of (X, μ) and (Y, λ) respectively such that M = P ∩ Q and N = P′ ∩ Q′. Then M × N = (P × P′) ∩ (Q ×Q′) holds. Hence M × N∈S-SLC(X × Y, μ × λ).

Similarly, the proofs of (2) and (3) follow from the definitions.

3.2 Supra Generalized Locally semi-closed and Supra Semi-locally Generalized Closed Sets

In this section, we introduce supra glsc sets and supra slgc sets and its some properties are obtained.

Definition 1.

Let (X, μ) be a supra topological space. A subset A of (X, μ) is called supra generalized locally semi-closed set (briefly supra glsc set), if A=U ∩ V, where U is supra g-open in (X, μ) and V is supra semi-closed in (X, μ).

The collection of all supra generalized locally semi-closed sets of X will be denoted by S-GLSC(X).

Remark 2.

Every supra semi-closed set (resp. supra g-open set) is S-GLSC.

Definition 3.

Let (X, μ) be a supra topological space. A subset A of (X, μ) is called supra semi locally generalized closed set (briefly supra slgc set), if A=U ∩ V, where U is supra semi-open in (X, μ) and V is supra g-closed in (X, μ).

The collection of all supra semi locally generalized closed sets of X will be denoted by S-SLGC(X).

Remark 4.

Every supra g-closed set (resp. supra semi-open set) is S-SLGC.

Definition 5.

A supra topological space (X, μ) is called a S-SG space, if the intersection of a supra semi-closed with a supra g-closed set is supra g-closed.

Example 6.

Let X = {a, b, c, d} and μ = {ϕ, X, {a, b, c}, {a, b, d}}. Then (X, μ) is S-SG space. Let X = {a, b, c, d} and μ = {ϕ, X, {a, c}, {b, c}, {a, b, c}, {a, b, d}, {b, c, d}}. This (X, μ) is not S-SG space.

Theorem 7.

Let A be a subset of (X, μ). Then A∈S-GLSC if and only if A = U ∩ $scl^{\mu}(A)$, for some supra g-open set U.

Proof.

Sufficiency: Let A = U ∩ $scl^{\mu}(A)$ where U is supra g-open. Then A is S-GLSC as $scl^{\mu}(A)$ is supra semi-closed.

Necessity: Let A ∈ S-GLSC(X, μ)

Then there exist a supra g-open subset U and a supra semi-closed subset V such that A=U ∩V. Since A ⊂ U and A ⊂ $scl^{\mu}(A)$, A ⊂ U ∩ $scl^{\mu}(A)$.

Conversely, we have $scl^{\mu}(A)$⊂ V and hence A = U ∩ V ⊃ U ∩ $scl^{\mu}(A)$. Therefore A = U ∩ $scl^{\mu}(A)$.

Theorem 8.

Let A be a subset of (X, μ). Then A∈S-SLGC if and only if A = U ∩ $cl_g^{\mu}(A)$, for some supra semi-open set U.

Proof.

Sufficiency: Let A = U ∩ $cl_g^{\mu}(A)$, where U is supra semi-open. Then A is S-SLGC as $cl_g^{\mu}(A)$ is supra g-closed.

Necessity: Let A ∈ S-SLGC(X, μ)

Then there exist a supra semi-open subset U and a supra g-closed subset V such that A=U ∩V. Since A ⊂ U and A ⊂ $cl_g^{\mu}(A)$, A ⊂ U ∩ $cl_g^{\mu}(A)$.

Conversely, we have $cl_g^{\mu}(A)$ ⊂ V and hence A = U ∩ V ⊃ U ∩ $cl_g^{\mu}(A)$. Therefore A = U∩ $cl_g^{\mu}(A)$.

Theorem 9.

For a subset A of a S-SG space (X, μ), the following are equivalent:

(i) A ∈ S-GLSC(X, μ).

(ii) $scl^{\mu}(A)$ – A is supra g-closed.

(iii) A∪ [X- $scl^{\mu}(A)$] is supra g-open.

Proof

(i) ⇒ (ii): By the theorem 7, A = U ∩ $scl^{\mu}(A)$, where U is supra g-open. Now $scl^{\mu}(A)$ – A = $scl^{\mu}(A)$ – U = $scl^{\mu}(A)$ ∩ (X-U) where $scl^{\mu}(A)$ is supra semi-closed and X-U is supra g-closed. Since the space is a S-SG spaces, $scl^{\mu}(A)$ ∩ (X-U) is supra g-closed. Thus $scl^{\mu}(A)$ – A is supra g-closed.

(ii) ⇒ (iii): X- [$scl^{\mu}(A)$ –A] = [X- $scl^{\mu}(A)$]∪A. Since $scl^{\mu}(A)$ –A is supra g-closed, X- [$scl^{\mu}(A)$ –A] is supra g-open. Therefore [X- $scl^{\mu}(A)$]∪A is supra g-open.

(iii) ⇒ (i): A= [X-[$scl^{\mu}(A)$-A]]∩ $scl^{\mu}(A)$. By (iii), X-[$scl^{\mu}(A)$-A] is supra g-open. It follows from theorem 7 that A is S-GLSC.

Theorem 10.

Let A and Z be subset of (X, μ) and let A ⊆ Z. If Z is supra g-open and supra preopen in (X, μ) and A ∈ S-GLSC (Z, μ/Z), then A ∈ S-GLSC(X, μ).

Proof.

By theorem 7, there exist a supra g-open set H of (Z, μ/Z) such that A = H ∩ $scl_{Z}^{\mu}(A)$. But $scl_{Z}^{\mu}(A)$ = Z ∩ $scl^{\mu}(A)$. Therefore A = H ∩ Z ∩ $scl^{\mu}(A)$, where H ∩ Z is supra g-open. Thus A ∈ S-SLC** (X, μ).

Remark 11.

(i) Intersection of two supra S-GLSC set is generally not an S-GLSC set.

(ii) Intersection of two supra S-SLGC set is generally not an S-SLGC set.

Example 12.

Let X = {a, b, c, d} and μ = {ϕ, X, {a, c}, {b, c}, {a, b, c}, {a, b, d}, {b, c, d}}.

Then

(i) {a, b, c} and {a, b, d} are S-SLGC but {a, b} not S-SLGC.

(ii) {a, b, c} and {a, b, d} are S-GLSC but {a, b} not S-GLSC.

Remark 13.

(i) Union of two supra S-GLSC set is generally not a S-GLSC set.

(ii) Union of two supra S-SLGC set is generally not an S-SLGC set.

3.3 Supra Semi Generalized Locally Closed Sets

In this we define Supra semi generalized locally closed sets and discuss some of their properties.

Definition 1.

Let (X, μ) be a supra topological space. A subset A of (X, μ) is called supra semi generalized locally closed set (briefly supra sg-locally closed set), if A=U ∩ V, where U is supra sg-open in (X, μ) and V is supra sg-closed in (X, μ).

The collection of all supra generalized locally closed sets of X will be denoted by S-SGLC(X).

Remark 2.

Every supra sg-closed set (resp. supra sg-open set) is S-SGLC.

Definition 3.

For a subset A of supra topological space (X, μ), A ∈ S-SGLC*(X,μ), if there exist a supra sg-open set U and a supra closed set V of (X,μ), respectively such that A=U ∩ V.

Remark 4.

Every supra closed set (resp. supra sg-open set) is S-SGLC*.

Definition 5.

For a subset A of (X, μ), A ∈ S-SGLC**(X, μ), if there exist a supra open set U and a supra sg-closed set V of (X, μ), respectively such that A=U ∩ V.

Remark 6.

Every supra sg-closed set (resp. supra open set) is S-SGLC**.

Definition 7.

Let A be a subset (X, μ). Then

(i) The supra sg-closure of a set A is, denoted by $cl_{sg}^{\mu}(A)$, defined as $cl_{sg}^{\mu}(A)$ = ∩ {B : B is a supra sg-closed and A ⊆ B}.

(ii) The supra sg-interior of a set A is, denoted by $int_{sg}^{\mu}(A)$, defined as $int_{sg}^{\mu}(A)$ = ∪ {B : B is a supra sg-open and B ⊆ A}.

Theorem 8.

For a supra topological space (X, μ), the following implications hold:

(i) S-LC(X, μ) ⊂ S-SLC**(X, μ) ⊂ S-SLC(X, μ) ⊂ S-SGLC(X, μ).

(ii) S-LC(X, μ) ⊂ S-SLC**(X, μ) ⊂ S-SGLC**(X, μ) ⊂ S-SGLC(X, μ).

(iii) S-LC(X, μ) ⊂ S-SGLC*(X, μ) ⊂ S-SGLC(X, μ).

(iv) S-LC(X, μ) ⊂ S-SLC**(X, μ) ⊂ S-GLSC(X, μ).

(v) S-LC(X, μ) ⊂ S-GLC*(X, μ) ⊂ S-GLSC(X, μ).

Remark 9.

The converses of none of the above are need not be true as seen from the following examples.

Example 10.

Let X = {a, b, c, d} and μ = {ϕ, X, {a, c}, {b, c}, {a, b, c}, {a, b, d}, {b, c, d}}.

Then (1) {a, c, d} is S-SLC but not S-LC and also not S-SLC*.

(2) {a, c, d} is S-SLC but not S-GLSC.

(3) {a, c, d} is S-SGLC but not S-LC.

Example 11

Let X = {a, b, c} and μ = {ϕ, X, {a, b, c}, {a, b, d}}.

(i) Then {b, c} is S-SGLC but not S-GLSC.

(ii) Then {a} is S-SGLC but not S-SLC.

(iii) Then {a} is S-GLC but not S-SLC.

(iv) Then {a, b} is S-GLSC but not S-SLC.

Example 12

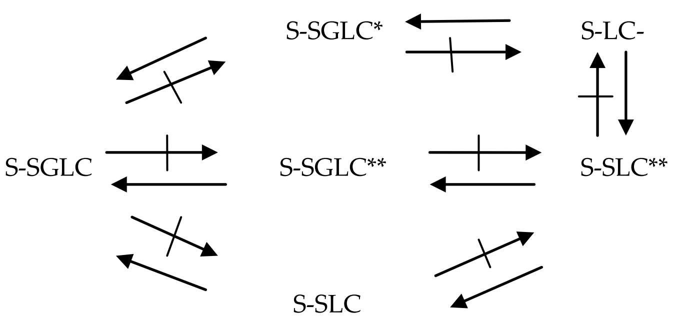
Let X = {a, b, c, d} and μ = { ϕ, X, {a, b, c}, {a, b, d}}. Then S-SGLC*(X, μ) = {ϕ, X, {a}, {b}, {c}, {d}, {a, b}, {a, b, c}, {a, b, d}} and S-SGLC**(X, μ) = {ϕ, X, {c}, {d}, {a, c}, {a, d}, {b, c}, {b, d} {a, d}, {a, b, c}, {a, b, d}, {a, c, d}, {b, c, d}}. Thus we conclude that S-SGLC* and S-SGLC** are independent.

Example 13.

Let X = {a, b, c, d} and μ = { ϕ, X, {a}, {a, b}, {b, c}, {a, b, c}}.Then the supra locally closed sets are ϕ, X, {a}, {b}, {c}, {d}, {a, b}, {b, c}, {c, d} {a, d}, {a, b, c}, {b, c, d}} and S-SGLC*(X, μ) = P(X)-{a, c} and {a, c, d}. Then S-SGLC**(X, μ) = {b, d} and {a, b, d}. S-SGLC(X, μ) = P(X). From this, a supra locally closed set is a proper subset of S-SGLC(X, μ).

Remark 14.

From the above, we have the following diagram



where A $\longrightarrow$ B (resp., A $\not\longrightarrow$ B) represents that A implies B (resp, A does not imply B).

Theorem 15.

For a subset A of (X, μ), the following are equivalent:

(iv) A ∈ S-SGLC*(X, μ).

(v) A = U ∩ $cl^{\mu}(A)$, for some supra sg-open set U.

(vi) $cl^{\mu}(A)$ – A is supra sg-closed.

(vii) A∪ [X- $cl^{\mu}(A)$], is supra sg-open.

Proof: (i) ⇒ (ii): Given A ∈ S-SGLC(X, μ)

Then there exist a supra sg-open subset U and a supra closed subset V such that A=U ∩V. Since A ⊂ U and A ⊂ $cl^{\mu}(A)$, A ⊂ U ∩ $cl^{\mu}(A)$.

Conversely, we have $cl^{\mu}(A)$⊂ V and hence A = U ∩ V ⊃ U ∩ $cl^{\mu}(A)$. Therefore A = U ∩ $cl^{\mu}(A)$

(ii) ⇒ (i): Let A = U∩ $cl^{\mu}(A)$, for some supra sg-open set U. Clearly, $cl^{\mu}(A)$ is supra sg-closed and hence A = U ∩ $cl^{\mu}(A)$∈ $\mathrm{S-RGLC(X, \mu)}$.

(ii) ⇒ (iii): Let A = U ∩ $cl^{\mu}(A)$, for some supra sg-open set U.

Then A ∈ S-SGLC (X, μ). This implies U is supra sg-open and $cl^{\mu}(A)$ is supra sg-closed. Therefore, $cl^{\mu}(A) - $A is supra sg-closed.

(iii) ⇒ (ii): Let U= X – [$cl^{\mu}(A)$- A]. By (iii), U is supra sg-open in X. Then A = U ∩ $cl^{\mu}(A)$ holds.

(iii) ⇒ (iv): Let Q =$cl^{\mu}(A)$–A be supra sg-closed. Then X-Q = X - [$cl^{\mu}(A)$– A] =A ∪ [(X- $cl^{\mu}(A)$]. Since X-Q is supra sg-open, A ∪ [X- $cl^{\mu}(A)$] is supra sg-open.

(vi) ⇒ (iii): Let U = A ∪ [(X- $cl^{\mu}(A)$]. Since X – U is supra sg-closed and X - U = $cl^{\mu}(A)$ - A holds, $cl^{\mu}(A)$ - A is supra sg-closed.

Theorem 16.

For a subset A of (X, μ), the following are equivalent:

(i) A ∈ S-SGLC(X, μ).

(ii) A = U ∩ $cl^{\mu}_{sg}(A)$, for some supra sg-open set U.

(iii) $cl^{\mu}_{sg}(A)$ - A is supra sg-closed.

(iv) A∪ [X - $cl^{\mu}_{sg}(A)$] is supra sg-open.

Proof. (i) ⇒ (ii): Given A ∈ S-SGLC(X, μ).

Then there exist a supra sg-open subset U and a supra sg-closed subset V such that A=U ∩V. Since A ⊂ U and A ⊂ $cl^{\mu}_{sg}(A)$, A ⊂ U ∩ $cl^{\mu}_{sg}(A)$.

Conversely by theorem 2.9 (iv), $cl^{\mu}_{sg}(A)$ ⊂ V and hence A = U ∩ V ⊃ U ∩ $cl^{\mu}_{sg}(A)$. Therefore, A = U ∩ $cl^{\mu}_{sg}(A)$.

(ii) ⇒ (i): Let A = U∩ $cl^{\mu}_{sg}(A)$, for some supra sg-open set U. Then we have, $cl^{\mu}_{sg}(A)$ is supra sg-closed and hence A = U ∩ $cl^{\mu}_{sg}(A)$ ∈ S-SGLC(X, μ).

(ii) ⇒ (iii): Let A = U ∩ $cl^{\mu}_{sg}(A)$, for some supra sg-open set U.

Then A ∈ S-SGLC (X, μ). This implies U is supra sg-open and $cl^{\mu}_{sg}(A)$ is supra sg-closed. Therefore, $cl^{\mu}_{sg}(A) - $A is supra sg-closed.

(iii) ⇒ (ii): Let U= X – [$cl^{\mu}_{sg}(A)$ - A]. By (iii), U is supra sg-open in X. Then A = U ∩ $cl^{\mu}_{sg}(A)$ holds.

(iii) ⇒ (iv): Let Q = $cl^{\mu}_{sg}(A)$–A be supra sg-closed. Then X-Q = X - [$cl^{\mu}_{sg}(A)$ – A] =A ∪ [(X- $cl^{\mu}_{sg}(A)$]. Since X-Q is supra sg-open, A ∪ [X- $cl^{\mu}_{sg}(A)$] is supra sg-open.

(vi) ⇒ (iii): Let U = A ∪ [(X- $cl^{\mu}_{sg}(A)$]. Since X – U is supra sg-closed and X - U = $cl^{\mu}_{sg}(A)$ - A is supra sg-closed.

Theorem 17.

For a subset A of (X, μ), if A ∈ S-SGLC**(X, μ), then there exist an supra open set G such that A = G ∩ $cl^{\mu}(A)$.

Proof. Let A ∈ S-SGLC**(X, μ). Then A=G ∩ V, where G is supra open set and V is supra sg-closed set. Then A =G ∩ V ⇒ A ⊂ G. Obviously, A ⊂ $cl^{\mu}(A)$. Thus A ⊂ G ∩ $cl^{\mu}(A)$ ----- (1)

Also we have $cl^{\mu}(A)$ ⊂ V. This implies A = G ∩ V ⊃ G ∩ $cl^{\mu}(A)$.

Therefore, A ⊃ G ∩ $cl^{\mu}(A)$ ----- (2). From (1) and (2), we get A = G ∩ $cl^{\mu}(A)$.

Theorem 18.

For a subset A of (X, μ), if A ∈ S-SGLC**(X, μ), then there exist an supra open set G such that A = G ∩ $cl_{sg}^{\mu}(A)$.

Proof. Let A ∈ S-SGLC**(X, μ). By the definition, A=G ∩ V, where G is supra open set and V is supra sg-closed set. Then A =G ∩ V ⇒ A ⊂ G. Since A ⊂ $cl_{sg}^{\mu}(A)$, A ⊂ G ∩ $cl_{sg}^{\mu}(A)$ ----- (1)

Also we have $cl_{sg}^{\mu}(A)$ ⊂ V. Then A = G ∩ V ⊃ G ∩ $cl_{sg}^{\mu}(A)$. Hence A ⊃ G ∩ $cl_{sg}^{\mu}(A)$ ----- (2). From (1) and (2), we get A = G ∩ $cl_{sg}^{\mu}(A)$.

Theorem 19.

Let A be a subset of (X, μ). If A ∈ S-SGLC**(X, μ), then $cl_{sg}^{\mu}(A)$ - A supra sg-closed and A ∪ [(X- $cl_{sg}^{\mu}(A)$] is supra sg-open.

Proof. The proof is obvious from the definitions and results.

Remark 20.

The converse of the above theorem need not be true as seen the following example.

Example 21.

Let X = {a, b, c, d} and μ = {ϕ, X, {a, c}, {b, c}, {a, b, c}, {a, b, d}, {b, c, d}}. Then ϕ, X, {a}, {b}, {c}, {d}, {a, d}, {b, d} are supra sg-closed sets in X and S-SGLC**(X, μ) = P(X) – {a, b}, {c, d} and {a, c, d}. If A = {a, c, d}, then $cl_{sg}^{\mu}(A)$ – A is supra sg-closed and A ∪ [(X - $cl_{sg}^{\mu}(A)$] is supra sg-open but A ∉ S-SGLC**(X, μ).

Remark 22.

(i) Intersection of two supra S-SGLC set is generally not an S-SGLC set.
(ii) Intersection of two supra S-SGLC* set is generally not an S-SGLC* set.
(iii) Intersection of two supra S-SGLC** set is generally not an S-SGLC** set.

Example 23.

Let X = {a, b, c, d} and μ = {ϕ, X, {a, c}, {b, c}, {a, b, c}, {a, b, d}, {b, c, d}}.

Then
(i) {a, c, d} and {b, c, d} are S-SGLC but {c, d} not S-SGLC.
(ii) {b, c, d} and {a, c, d} are S-SGLC* but {c, d} not S-SGLC*.
(iii) {a, b, c} and {a, b, d} are S-SGLC** but {a, b} not S-SGLC**.

Remark 24.

(i) Union of two supra S-SGLC set is generally not an S-SGLC set.
(ii) Union of two supra S-SGLC* set is generally not an S-SGLC* set.
(iii) Union of two supra S-SGLC** set is generally not an S-SGLC** set.

Example 25.

Let X = {a, b, c, d} and μ = {ϕ, X, {a, c}, (b, c}, {a, b, c}, {a, b, d}, {b, c, d}}.

Then
(i) {c} and {d} are S-SGLC but {c, d} not S-SGLC.
(ii) {a} and {b} are S-SGLC* but {a, b} not S-SGLC*.
(iii) {d} and {a, c} are S-SGLC** but {a, c, d} not S-SGLC**.

Theorem 26.

Let A and Z be subset of (X, μ) and let A ⊆ Z. If Z is supra sg-open in (X, μ) and A ∈ S-SGLC* (Z, μ/Z), then A ∈ S-SGLC* (X, μ).

Proof. Suppose A ∈ S-SGLC*, then there exist a supra sg-open set H of (Z, μ/Z) such that A = H ∩ $cl_{Z}^{\mu}(A)$. But $cl_{Z}^{\mu}(A)$ = Z ∩ $cl^{\mu}(A)$. Therefore A = H ∩ Z ∩ $cl^{\mu}(A)$, where H ∩ Z is supra sg-open. Thus A ∈ S-SGLC* (X, μ).

Remark 27.

The following example shows that the assumption that Z is supra sg-open cannot be removed from the above theorem.

Example 28.

Let X = {a, b, c, d}, μ = {ϕ, X, {a, b}, {b, d}, {a, b, d}, {a, c, d}}. Let V be the collection of all supra sg-open sets of (X, μ). Then V = {ϕ, X, {a, b}, {b, d}, {a, b, c}, {a, b, d}, {a, c, d}, {b, c, d}}. Put Z = A = {a, d}. Then Z is not supra sg-open and A∈ S-SGLC* (Z, μ/Z). However A∉ S-SGLC* (X, μ), since S-SGLC* (X, μ) = P(X) – {a, d} and {b, c}.

Theorem 29.

Let {Z_i / i∈ Ω} be a cover of (X, μ), where Ω is a finite set and let A be a subset of (X, μ). Suppose that Z_i [i∈ Ω] are supra pre-open, supra semi-open and supra sg-closed sets of (X, μ) is closed under finite unions. If A ∩ Z_i ∈ S-SGLC** (Z_i, μ|Z_i) for every i∈ Ω, then A∈ S-SGLC** (X, μ).

Proof. Let i∈Ω. Since A ∩ Z_i ∈ S-SGLC** (Z_i, μ|Z_i), there exist a supra open set V_i∈μ and supra sg-closed set G_i of (Z_i, μ| Z_i) such that A ∩ Z_i = V_i ∩ G_i ∩ Z_i = V_i∩ (G_i∩ Z_i). Then A = ∪ { A ∩ Z_i / i∈ Ω} = ∪ {{ V_i / i∈ Ω} ∩ [∪{ Z_i∩ G_i / i∈Ω}]}. This shows that A∈ S-SGLC** (X, μ).

Theorem 30.

Let (X, μ) and (Y, λ) be the supra topological spaces.

(4) If M ∈ S-SGLC(X, μ) and N ∈ S-SGLC(Y, λ), then M × N∈S-SGLC(X × Y, μ × λ).
(5) If M ∈ S-SGLC*(X, μ) and N ∈ S-SGLC*(Y, λ), then M × N∈S-SGLC*(X × Y, μ × λ).
(6) If M ∈ S-SGLC**(X, μ) and N ∈ S-SGLC**(Y, λ), then M × N ∈ S-SGLC**(X ×Y, μ × λ).

Proof. Let M ∈ S-SGLC(X, μ) and N ∈ S-SGLC(Y, λ). Then there exist a supra sg-open sets P and P′ of (X, μ) and (Y, λ) and supra sg-closed sets Q and Q′ of (X, μ) and (Y, λ) respectively such that M = P ∩ Q and N = P′ ∩ Q′. Then M × N = (P × P′) ∩ (Q ×Q′) holds. Hence M × N∈S-SGLC(X × Y, μ × λ).

Similarly, the proofs of (2) and (3) follow from the definitions.

4. Weaker Forms of Supra Locally Closed Continuous Functions

In this section we define some functions which are weaker than supra locally closed continuous function and study the relationship between them.

4.1 Supra Semi Locally Continuous Functions

In this we introduce a functions called supra semi locally continuous functions and supra semi locally irresolute functions. Also investigate its properties.

Definition 1.

Let (X, τ) and (Y, σ) be two topological spaces and $\tau \subseteq \mu$. A function $f: (X, \tau) \to (Y,\sigma)$ is called S-SL-continuous (resp., S-SL* - continuous, resp., S-SL** - continuous), if $f^{-1}(A) \in$ S-SLC (X,μ), (resp., $f^{-1}(A) \in$ S-SLC* (X,μ), resp., $f^{-1}(A) \in$ S-SLC** (X,μ)) for each $A \in \sigma$.

Definition 2.

Let (X, τ) and (Y, σ) be two topological spaces and μ and λ be a supra topologies associated with τ and σ respectively. A function $f : (X, \tau) \to (Y,\sigma)$ is said to be S-SL – irresolute (resp., S-SL* - irresolute, resp., S-SL** - irresolute) if $f^{-1}(A) \in$ S-SLC (X,μ), (resp., $f^{-1}(A) \in$ S-SLC* (X,μ), resp., $f^{-1}(A) \in$ S-SLC**(X,μ)) for each $A \in$ S-SLC (Y, λ) (resp., $A \in$ S-SLC* (Y, λ), resp., $A \in$ S-SLC**(Y, λ)).

Theorem 3.

Let (X, τ) and (Y, σ) be two topological spaces and μ and λ be a supra topologies associated with τ and σ respectively. Let $f: (X, \mu) \to (Y,\sigma)$ be a function.

(i) If f is supra locally closed continuous, then it is S-SL-continuous.

(ii) If f is supra irresolute and supra continuous, then it is S-SL-irresolute.

(iii) If f is supra irresolute, then it is S-SL-continuous.

Proof. By the definitions the proof is immediate.

Example 4.

Let $X = Y = \{a, b, c, d\}$ with $\tau =\{\phi, X, \{a, b, c\}\}$, $\sigma = \{\phi, Y, \{a, c, d\}\}$ and $\mu = \{\phi, X, \{a, c\}, \{b, c\}, \{a, b, c\}, \{a, b, d\}, \{b, c, d\}\}$. Define $f : (X,\mu) \to (Y,\sigma)$ is an identity function. Then f is S-SL- continuous, but it is not S-LC-continuous.

Theorem 5.

If $g: X \to Y$ is S-SL- continuous and $h: Y \to Z$ is supra continuous, then $hog: X \to Z$ is S-SL – continuous.

Proof. Let $g: X \to Y$ is S-SL – continuous and $h : Y \to Z$ is supra continuous. By the definitions, $g^{-1}(V) \in$ S-SLC (X) , $V \in Y$ and $h^{-1}(W) \in Y$, $W \in Z$. Let $W \in Z$. Then $(hog)^{-1}(W) = (g^{-1} h^{-1})(W)= g^{-1}(h^{-1}(W))=g^{-1}(V)$, for $V \in Y$. Thus $(hog)^{-1}(W) = g^{-1}(V) \in$ S-SLC (X), $W \in Z$. Therefore, hog is S-SL-continuous.

Theorem 6.

If $g: X \to Y$ is S-SL – irresolute and $h: Y \to Z$ is S-SL-continuous, then $h \circ g : X \to Z$ is S-SL – continuous.

Proof. Let $g : X \to Y$ is S-SL – irresolute and $h : Y \to Z$ is S-SL-continuous. By the definitions, $g^{-1}(V) \in$ S-SLC (X) , for $V \in$S-SLC (Y) and $h^{-1}(W) \in$S-SLC (Y) , for $W \in Z$. Let $W \in Z$. Then $(hog)^{-1}(W) = (g^{-1} h^{-1}) (W) = g^{-1}(h^{-1}(W)) = g^{-1}(V)$, for $V \in$ S-SLC(Y). $(hog)^{-1}(W) = g^{-1}(V) \in$ S-SLC (X), $W \in Z$. Hence hog is S-SL- continuous.

Theorem 7.

If $g : X \to Y$ and $h : Y \to Z$ are S-SL-irresolute, then $h \circ g : X \to Z$ is also S-SL – irresolute.

Proof. By the hypothesis and the definitions, we have $g^{-1}(V) \in$ S-SLC(X), for $V \in$S-SLC(Y) and $h^{-1}(W) \in$S-SLC(Y), for $W \in$ S-SLC(Z). Let $W \in$S-SLC(Z). Then $(hog)^{-1}(W) = (g^{-1} h^{-1}) (W) = g^{-1}(h^{-1}(W)) = g^{-1}(V)$, for $V \in$ S-SLC(Y). $(hog)^{-1}(W) = g^{-1}(V) \in$ S-SLC (X), $W \in$ S-SLC(Z). Thus hog is S-SL – irresolute.

4.2 Supra Generalized Locally Semi Continuous Functions and Supra Semi locally Generalized Continuous functions

In this section we study the S-GLS-continuous Functions and S-SLG-continuous functions and also their respective irresolute functions.

Definition 1.

Let (X, τ) and (Y, σ) be two topological spaces and $\tau \subseteq \mu$. A function $f: (X, \tau) \to (Y,\sigma)$ is called S-GLS-continuous, if $f^{-1}(A) \in$ S-GLSC (X,μ), for each $A \in \sigma$.

Definition 2.

Let (X, τ) and (Y, σ) be two topological spaces and $\tau \subseteq \mu$. A function $f: (X, \tau) \to (Y,\sigma)$ is called S-SLG-continuous, if $f^{-1}(A) \in$ S-SLGC (X,μ), for each $A \in \sigma$.

Definition 3.

Let (X, τ) and (Y, σ) be two topological spaces and $\tau \subseteq \mu$ and $\sigma \subseteq \lambda$. A function $f: (X, \tau) \to (Y,\sigma)$ is called S-GLS- irresolute, if $f^{-1}(A) \in$ S-GLSC (X, μ), for each $A \in$ S-GLSC(Y,λ).

Definition 4.

Let (X, τ) and (Y, σ) be two topological spaces and $\tau \subseteq \mu$ and $\sigma \subseteq \lambda$. A function $f: (X, \tau) \to (Y,\sigma)$ is called S-SLG- irresolute, if $f^{-1}(A) \in$ S-SLGC (X, μ), for each $A \in$ S-SLGC(Y, λ).

Theorem 5.

If $g: X \to Y$ is S-GLS- continuous and $h: Y \to Z$ is supra continuous, then $hog: X \to Z$ is S-SL – continuous.

Proof. Let $g: X \to Y$ is S-GLS-continuous and $h : Y \to Z$ is supra continuous. By the definitions, $g^{-1}(V) \in$ S-GLSC (X) , $V \in Y$ and $h^{-1}(W) \in Y$, $W \in Z$. Let $W \in Z$. Then $(hog)^{-1}(W) = (g^{-1} h^{-1})(W)= g^{-1}(h^{-1}(W))=g^{-1}(V)$, for $V \in Y$. Thus $(hog)^{-1}(W) = g^{-1}(V) \in$ S-GLSC (X), $W \in Z$. Therefore, hog is S-GLS-continuous.

Theorem 6.

If $g: X \to Y$ is S-GLS– irresolute and $h: Y \to Z$ is S-GLS-continuous, then $h \circ g : X \to Z$ is S-GLS-continuous.

Proof. Let $g : X \to Y$ is S-GLS-irresolute and $h : Y \to Z$ is S-GLS-continuous. By the definitions, $g^{-1}(V) \in$ S-GLSC (X) , for $V \in$S-GLSC (Y) and $h^{-1}(W) \in$S-GLSC (Y) , for $W \in Z$. Let $W \in Z$. Then $(hog)^{-1}(W) = (g^{-1} h^{-1}) (W) = g^{-1}(h^{-1}(W)) = g^{-1}(V)$, for $V \in$ S-GLSC(Y). $(hog)^{-1}(W) = g^{-1}(V) \in$ S-GLSC (X), $W \in Z$. Hence hog is S-GLS-continuous.

Theorem 7.

If g: X $\rightarrow$ Y and h : Y $\rightarrow$ Z are S-GLS-irresolute, then h o g : X $\rightarrow$ Z is also S-GLS-irresolute.

Proof: By the hypothesis and the definitions, we have $g^{-1}(V) \in$ S-GLSC(X), for V $\in$S-GLSC(Y) and $h^{-1}(W) \in$S-GLSC(Y), for W$\in$ S-GLSC(Z). Let W $\in$S-GLSC(Z). Then $(hog)^{-1}(W) = (g^{-1} h^{-1})(W) = g^{-1}(h^{-1}(W)) = g^{-1}(V)$, for V $\in$ S-GLSC(Y). $(hog)^{-1}(W) = g^{-1}(V) \in$ S-GLSC (X), W $\in$ S-GLSC (Z). Thus hog is S-GLS-irresolute.

Theorem 8

If g: X $\rightarrow$ Y is S-SLG- continuous and h: Y $\rightarrow$ Z is supra continuous, then hog: X $\rightarrow$ Z is S-SL – continuous.

Proof. Let g: X $\rightarrow$ Y is S-SLG-continuous and h : Y $\rightarrow$ Z is supra continuous. By the definitions, $g^{-1}(V) \in$ S-SLGC (X) , V $\in$Y and h^{-1} (W) $\in$Y , W$\in$ Z. Let W $\in$ Z. Then $(hog)^{-1}(W) = (g^{-1} h^{-1})(W)= g^{-1}(h^{-1}(W))=g^{-1}(V)$, for V $\in$ Y. Thus $(hog)^{-1}(W) = g^{-1}(V) \in$ S-SLGC (X), W $\in$ Z. Therefore, hog is S-SLG-continuous.

Theorem 9.

If g: X $\rightarrow$ Y is S-SLG– irresolute and h: Y $\rightarrow$ Z is S-SLG-continuous, then h o g : X $\rightarrow$ Z is S-SLG-continuous.

Proof. Let g : X $\rightarrow$ Y is S-SLG-irresolute and h : Y $\rightarrow$ Z is S-SLG-continuous. By the definitions, $g^{-1}(V) \in$ S-SLGC (X) , for V $\in$S-SLGC (Y) and $h^{-1}(W) \in$S-SLGC (Y) , for W$\in$ Z. Let W $\in$ Z. Then $(hog)^{-1}(W) = (g^{-1} h^{-1})(W) = g^{-1}(h^{-1}(W)) = g^{-1}(V)$, for V $\in$ S-SLGC(Y). $(hog)^{-1}(W) = g^{-1}(V) \in$ S-SLGC (X), W $\in$ Z. Hence hog is S-SLG-continuous.

Theorem 10.

If g: X $\rightarrow$ Y and h : Y $\rightarrow$ Z are S-SLG-irresolute, then h o g : X $\rightarrow$ Z is also S-SLG-irresolute.

Proof: By the hypothesis and the definitions, we have $g^{-1}(V) \in$ S-SLGC(X), for V $\in$S-SLGC(Y) and $h^{-1}(W) \in$S-SLGC(Y), for W$\in$ S-SLGC(Z). Let W $\in$S-SLGC(Z). Then $(hog)^{-1}(W) = (g^{-1} h^{-1})(W) = g^{-1}(h^{-1}(W)) = g^{-1}(V)$, for V $\in$ S-SLGC(Y). $(hog)^{-1}(W) = g^{-1}(V) \in$ S-SLGC (X), W $\in$ S-SLGC (Z). Thus hog is S-SLG-irresolute.

4.3 Supra Semi Generalized Locally Continuous Functions

In this section the notion of supra semi generalized locally continuous functions, supra semi generalized locally irresolute functions are introduced and their basic properties are investigated and obtained.

Definition 1.

Let (X, τ) and (Y, σ) be two topological spaces and $\tau \subseteq \mu$. A function f: (X, τ) $\rightarrow$ (Y,σ) is called S-SGL-continuous (resp., S-SGL* - continuous, resp., S-SGL** - continuous), if $f^{-1}(A) \in$ S-SGLC (X,μ), (resp., $f^{-1}(A) \in$ S-SGLC* (X,μ), resp., $f^{-1}(A) \in$ S-SGLC** (X,μ)) for each A $\in \sigma$.

Definition 2.

Let (X, τ) and (Y, σ) be two topological spaces and μ and λ be a supra topologies associated with τ and σ respectively. A function f : (X, τ) $\rightarrow$ (Y,σ) is said to be S-SGL – irresolute (resp., S-SGL* - irresolute, resp., S-SGL** - irresolute) if $f^{-1}(A) \in$ S-SGLC (X,μ), (resp., $f^{-1}(A) \in$ S-SGLC* (X,μ), resp., $f^{-1}(A) \in$ S-SGLC**(X,μ)) for each A $\in$ S-SGLC (Y, λ) (resp., A$\in$ S-SGLC* (Y, λ), resp., A $\in$ S-SGLC**(Y, λ)).

Theorem 3.

Let (X, τ) and (Y,σ) be two topological spaces and μ be a supra topology associated with τ. Let f: (x, τ) $\rightarrow$ (y,σ) be a function.

(i) If f is S-LC-continuous, then it is S-SL-continuous.

(ii) If f is S-SL-continuous, then it is S-SGL-continuous.

(iii) If f is S-LC-continuous, then it is S-SGL*-continuous.

(iv) If f is S-SL*-continuous, then it is S-SL-continuous and S-SGL**-continuous.

(v) If f is S-SGL*– continuous or S-SGL** - continuous, then it is S-SGL - continuous.

Proof. The proof is trivial from the definitions.

Theorem 4.

Let (X, τ) and (Y, σ) be two topological spaces and μ and λ be a supra topologies associated with τ and σ respectively. Let f: (X, μ) $\rightarrow$ (Y,σ) be a function. If f is S-SGL-irresolute (respectively S-SGL*-irresolute, respectively S-SGL**-irresolute), then it is S-SGL-continuous. (respectively S-SGL*-continuous, respectively S-SGL**-continuous).

Proof. By the definitions the proof is immediate.

Remark 5.

Converses of above theorems need not be hold as seen from the following examples.

Example 6.

Let X = Y = {a, b, c, d }with τ ={ϕ, X, {a, b, c}}, σ = {{ϕ, Y, {a, b}} and μ = {ϕ, X, {a, b, c}, {a, b, d}}. Let f : (X,μ) $\rightarrow$ (Y,σ) be the identity map. Here f is not S-SL-continuous, but it is S-SGL- continuous.

Example 7.

Let X = Y = {a, b, c, d }with τ ={ϕ, X, {a, b}, {b, d}}, σ = {{ϕ, Y, {a}, {a, b, c}} and μ = {ϕ, X, {a, b}, {b, d}, {a, b, d}, {a, c, d}}. Let f : (X,μ) $\rightarrow$ (Y,σ) be the identity map. Here f is not S-SL**-continuous, but it is S-SL- continuous. Also f is not S-SL**-continuous, but it is S-SL* - continuous.

Example 8.

Let X = Y = {a, b, c, d }with τ ={ϕ, X, {a, b}, {b, d}}, σ = {{ϕ, Y, {b, d}, {a, b, d}} and μ = {ϕ, X, {a}, {a, b}, {b, c}, {a, b, c}}. Let f : (X, μ) $\rightarrow$ (Y, σ) be the identity map. Here f is not S-SGL**-continuous, but it is S-SGL*- continuous.

Example 9.

Let X = Y = {a, b, c, d }with τ ={ϕ, X, {a, b}, {b, c}}, σ = {{ϕ, Y, {a}, {a, c, d}} and μ = {ϕ, X, {a}, {a, b}, {b, c}, {a, b, c}}. Let f : (X, μ) $\rightarrow$ (Y, σ) be the identity map. S-SGLC (X, μ) = P(X), S-SGLC*(X, μ) = P(X)-{a, c} and {a, c, d} and S-SGLC**(X, μ) = P(X) – {b, d} and{a, b, d}.

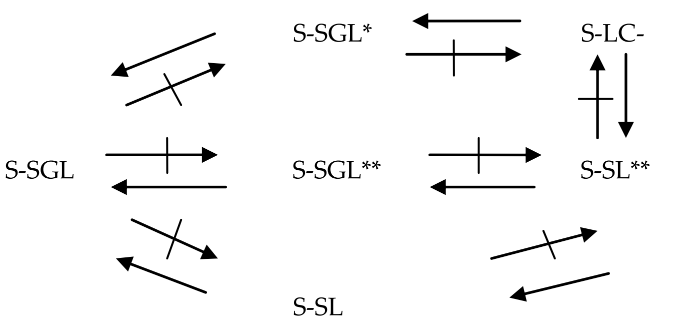
Here f is not S-SGL*– continuous, but it is S-SGL-continuous. Also f is not S-SGL*-continuous, but it is S-SGL** - continuous.

Example 10.

Let X = Y = {a, b, c, d }with τ = {ϕ, X, {a, b, c}}, σ = {{ϕ, Y, {b, d}, {a, b, d}}, μ = {ϕ, X, {a, b, c}, {a, b, d}} and λ = {ϕ, X, {a, b}, {b, d}, {a, b, d}, {a, c, d}}. Define f: (X,μ) $\to$ (Y,σ) by f(a) = f(b) = a, f(c) = d and f(d) = b. Then f is not S-SL-irresolute (respectively S-SL*-irresolute, respectively S-SL**-irresolute), but it is S-SL-continuous. (respectively S-SL*-continuous, respectively S-SL**-continuous).

Remark 11.

From the above results and examples we have the following implications.



where A $\longrightarrow$ B (resp., A $\not\longrightarrow$ B) represents that A implies B (resp, A does not imply B).

Theorem 12.

If g: X $\to$ Y is S-SGL- continuous and h: Y $\to$ Z is supra continuous, then hog: X $\to$ Z is S-SGL – continuous.

Proof. Let g: X $\to$ Y is S-SGL-continuous and h : Y $\to$ Z is supra continuous. By the definitions, $g^{-1}(V) \in$ S-SGLC (X) , V $\in$Y and h^{-1} (W) $\in$Y , W$\in$ Z. Let W $\in$ Z. Then $(hog)^{-1}(W) = (g^{-1} h^{-1})(W) = g^{-1}(h^{-1}(W)) = g^{-1}(V)$, for V $\in$ Y. Thus $(hog)^{-1}(W) = g^{-1}(V) \in$ S-SGLC (X), W $\in$ Z. Therefore, hog is S-SGL-continuous.

Theorem 13.

If g: X $\to$ Y is S-SGL – irresolute and h: Y $\to$ Z is S-SGL-continuous, then h o g: X $\to$ Z is S-SGL – continuous.

Proof. Let g: X $\to$ Y is S-SGL-irresolute and h: Y $\to$ Z is S-SGL-continuous. By the definitions, $g^{-1}(V) \in$ S-SGLC (X) , for V $\in$S-SGLC (Y) and $h^{-1}(W) \in$S-SGLC (Y) , for W$\in$ Z. Let W $\in$ Z. Then $(hog)^{-1}(W) = (g^{-1} h^{-1}) (W) = g^{-1}(h^{-1}(W)) = g^{-1}(V)$, for V $\in$ S-SGLC(Y). $(hog)^{-1}(W) = g^{-1}(V) \in$ S-SGLC (X), W $\in$ Z. Hence hog is S-SGL- continuous.

Theorem 14.

If g : X $\to$ Y and h : Y $\to$ Z are S-SGL – irresolute , then h o g : X $\to$ Z is also S-SGL- irresolute.

Proof. By the hypothesis and the definitions, we have $g^{-1}(V) \in$ S-SGLC(X), for V $\in$S-SGLC(Y) and $h^{-1}(W) \in$S-SGLC(Y), for W$\in$ S-SGLC(Z). Let W $\in$S-SGLC(Z). Then $(hog)^{-1}(W) = (g^{-1} h^{-1}) (W) = g^{-1}(h^{-1}(W)) = g^{-1}(V)$, for V $\in$ S-SGLC(Y). $(hog)^{-1}(W) = g^{-1}(V) \in$ S-SGLC (X), W $\in$ S-SGLC(Z). Thus hog is S-SGL-irresolute.

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